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The largest group of any given number of letters contains all the substitutions which correspond to the permutations of these letters. The number of substitutions in this group is therefore $n!$ and all the other groups of the same number of letters are subgroups of it. The second group, with respect to size contains all the substitutions which correspond to an even number of interchanges of letters, the number of substitutions in this group is $n! \div 2$.

[To be continued.]

NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

By GEORGE BRUCE HALSTED, A. M., (Princeton), Ph. D., (Johns Hopkins); Member of the London Mathematical Society; and Professor of Mathematics in the University of Texas, Austin, Texas.

[Continued from the April Number.]

PROPOSITION XVII. *In the hypothesis of acute angle, we can find a perpendicular and an oblique to the same straight which never meet.*

More literally: *If the straight AH stands* (fig. 15.) *at right angles to any one certain straight AB however small: I say that in the hypothesis of acute angle it cannot hold good, that every straight BD, making with AB any acute angle you choose, toward the parts of this AH, will at length meet this AH produced at a finite, or terminated distance.*

PROOF. Join HB. The angle ABH will be acute (Eu. I. 17), because of the right angle at the point A. Now (Eu. I. 23) a certain HD can be drawn toward the parts of the point B, which not cutting the angle AHB makes with this HB an acute angle equal to this acute angle ABH. Then from the point B is let fall to HD the perpendicular BD, which falls toward the parts of the aforesaid acute angle at the point H. Since therefore the side HB is opposite in the triangle HDB to the right angle at D, and likewise in the triangle BAH to the right angle at A; and again in those two triangles equal angles are adjacent to this side HB, which are in the first triangle indeed the angle BHD, and in the latter the angle HBA; also (Eu. I. 26) the remaining angle HBD in the former triangle will be equal to the remaining angle BHA in the latter triangle. Wherefore the entire angle DBA will be equal to the entire angle AHD. Now however, neither of the aforesaid equal angles will be obtuse, lest we meet (from the preceding proposition) in one case the now rejected hypothesis of obtuse angle. Nor will

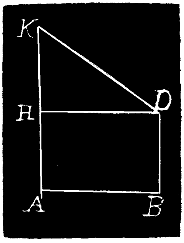

Fig. 15.

either be right, lest we meet (from the same preceding) in one case accordingly the hypothesis of right angle, which (P.V.) will leave no place for the hypothesis of acute angle. Therefore each one of those angles will be acute. This being the case; that the straight BD produced cannot meet in a certain point K this AH produced toward the same parts, is demonstrated thus; because in the triangle KDH, besides the right angle at D, is present the obtuse angle at H, since the angle AHD in the aforesaid hypothesis of acute angle is proved acute: But this is absurd, against Eu. I. 17. Therefore it cannot hold good in this hypothesis, that any BD, making with one straight AB as small as you choose, any acute angle towards the parts of this AH, will at length at a finite, or terminated distance, meet this AH produced. Quad erat demonstrandum.

The same otherwise and more easily. Two perpendiculars AK, BM stand on one certain small at will straight AB (fig. 16). From any point M of this BM let fall the perendicular MH, and join BH. It follows that the angle BHM will be acute. In the hypothesis of acute angle, the angle BMH is also (from the preceding proposition) acute. Therefore the perpendicular BDX, let fall from the point B to this HM, will cut (by Eu. I. 17) this HM in some intermediate point D. Therefore the angle XBA will be acute. But it follows (from the same Eu. I. 17) that those two straights AHK, BDX howsoever produced cannot mutually meet (anyhow at a finite, or terminated distance) on account of the right angles at the points H and D. Therefore in the hypothesis of acute angle it cannot hold good, that any BD, making with one however small straight AB any acute angle toward the parts of this AH, perpendicular to this same AB, will at length meet (at a finite, or terminated distance) this AH produced. Quad erat propositum.

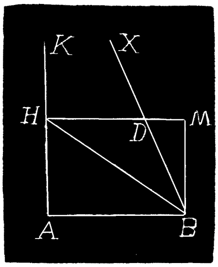

Fig. 16.

Scholion i. And this is, what I promised in the Scholia after proposition XIII, that the hypothesis of acute angle (which alone is able now to stand against that general Postutatum of Euclid) will certainly be destroyed by the sole admission of a general meeting of two straights toward those parts, toward which any straight, as small as you choose, meeting them, makes two internal angles less than two right angles; and so indeed, even if either of those angles is to be supposed right.

Scholion ii. But again in a better place, after proposition XXV, I will show that the hypothesis of acute angle will be equally destroyed, provided that any one acute angle as small as you choose can be designated, under which if any straight meets another, this produced must (at a finite, or terminated distance) finally meet any perpendicular erected upon this meeting straight at whatever finite distance.

Proposition xviii. *From any triangle ABC, of which* (fig. 17.) *the angle at the point B is inscribed in any semicircle, of which the diameter is AC, is established the hypothesis of right angle, or obtuse angle, or acute angle, according as indeed the angle at the point B was right, or obtuse, or acute.*

PROOF. From the center D join DB. The angles at the base AB will be (Eu. I. 5) equal, and likewise at the base BC, in the triangles ADB, CDB. Wherefore in the triangle ABC the two angles at the base AC will be together equal to the whole angle ABC. Therefore the three angles of the triangle ABC will be together equal to, or greater, or less than two right angles according as the angle at the point B was right, or obtuse, or acute. Therefore from any triangle ABC, of which the angle at the point B is inscribed in any semicircle, whose diameter is AC, is established (P.XV) the hypothesis of right angle, or obtuse angle, or acute angle, according as indeed the angle at the point B is right, or obtuse, or acute. Quod erat demonstrandum.

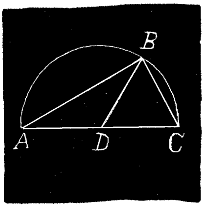

Fig. 17.

[To be continued]

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AN ATTEMPT TO DEMONSTRATE THE 11th AXIOM OF PLAYFAIR'S EUCLID.

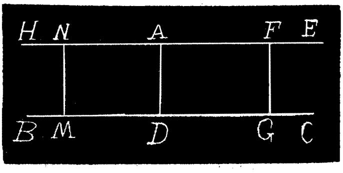
By WARREN HOLDEN, Professor of Mathematics. Girard College, Philadelphia, Pennsylvania.

"*Through a given point one line, and only one, can be drawn parallel to a given line.*"

1st. Two lines perpendicular to a third never intersect, how far soever they be produced. *Halsted's Lobatschewsky's Geometry.* Page 12. Art. 4.

2d. Parallel straight lines are such as are in the same plane, and which, being produced ever so far both ways, do not meet.

3d. Paralles are everywhere equidistant. From A draw AD perpendicular to BC, and through A draw HAE perpendicular to AD. AE and DC being perpendicular to the same line AD are parallel (1st and 2d). From *any* point F let fall the perpendicular FG upon BC. Lay off DM equal to DG, an erect the perpendicular MN. Fold over the part of the figure to the right of AD upon AD as an axis until it falls upon the part to the left. Since A, D, G and M are right angles, and since DM equals DG by construction, AF must fall upon AN, and GF upon MN. The point F is found upon AN and MN, at their intersection N. Therefore GF equals MN. Since F is *any* point, the parallels are everywhere equidistant.



4th. Through a given point one line, and only one, can be drawn parallel